Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Paper reference **8FM0/25**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
25: Further Mechanics 1
(Part of options C, E, H and J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

1. A car of mass 1200 kg moves up a straight road that is inclined to the horizontal at an angle α, where $\sin\alpha = \frac{1}{15}$

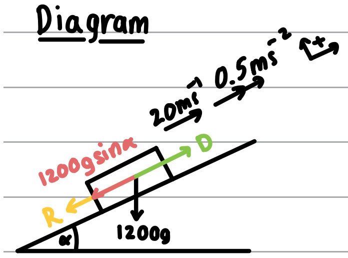
The total resistance to the motion of the car from non-gravitational forces is modelled as a constant force of magnitude R newtons.

At the instant when the engine of the car is working at a rate of 32 kW and the speed of the car is $20\,\text{m}\,\text{s}^{-1}$, the acceleration of the car is $0.5\,\text{m}\,\text{s}^{-2}$

Find the value of R

(5)

Diagram



To get D we will use Power.

Formula for Power:

Power (W) — $P = Dv$ (D: Driving force (N); v: velocity (ms^{-1}))

$P = 32\text{kW} \xrightarrow{\times 1000} 32\,000\text{W}$

$D = D$ N

$v = 20\text{ms}^{-1}$

Substitute: $32\,000 = 20D$

$D = 1600\text{N}$ M1

Since the car is accelerating, use $\Sigma F_x = ma$

$D - R - 1200g\sin\alpha = 1200(0.5)$ M1A1

$1600 - R - 1200g\left(\frac{1}{15}\right) = 600$ dM1

$1600 - 80g - 600 = R$

$1000 - 80(9.8) = R$

$1000 - 784 = 216$

$\therefore$ $R = 216\text{ N}$ value of R A1

Question 1 continued

(Total for Question 1 is 5 marks)

2. Two particles, A and B, have masses m and $3m$ respectively. The particles are moving in opposite directions along the same straight line on a smooth horizontal plane when they collide directly.

Immediately before they collide, A is moving with speed $2u$ and B is moving with speed u.

The direction of motion of each particle is reversed by the collision.

In the collision, the magnitude of the impulse exerted on A by B is $\frac{9mu}{2}$

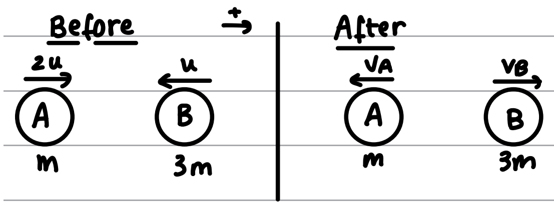
(a) Find the value of the coefficient of restitution between A and B.

(7)

(b) Hence, write down the total loss in kinetic energy due to the collision, giving a reason for your answer.

(1)

(a) Diagram



— we're given that the particles change direction, ∴ assume that in your diagram!

We can use the conservation of linear momentum to get this. M1

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (u: initial velocity, v: final velocity)

Substitute:

$m(2u) + 3m(-u) = m(-v_A) + 3m(v_B)$ cancel m's A1

$2u - 3u = -v_A + 3v_B$

$-u = 3v_B - v_A$ Eq.1

We can use Newton's Law of Restitution to get an equation. M1

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$ (e: coefficient of restitution, u: initial speed, v: final speed)

Substitute: (we want this!)

$e(2u - (-u)) = v_B - (-v_A)$

$3eu = v_A + v_B$ Eq.2

DO NOT WRITE IN THIS AREA

Question 2 continued

Solve simultaneously Eq1 and Eq2:

$$-u = 3v_B - v_A$$
$$3eu = v_B + v_A \quad +$$

use elimination method.

$$3eu - u = 4v_B$$
$$\frac{u}{4}(3e-1) = v_B$$

Speed of B after.

Formula for Impulse:

$$I = m(v - u)$$

velocity after (v), velocity before (u)

Let's use particle B since the magnitude of the impulse is the same for both particles.

Substitute:

$$I = 3m\left(\frac{u}{4}(3e-1) - (-u)\right)$$ M1A1

$$\frac{9mu}{2} = 3m\left(\frac{3eu}{4} - \frac{1}{4}u + u\right)$$ cancel m's

$$\frac{9u}{2} = 3\left(\frac{3eu}{4} + \frac{3u}{4}\right)$$ cancel u's A1

$$\left(\frac{18}{4}\right) \leftarrow \frac{9}{2} = \frac{9e}{4} + \frac{9}{4}$$

↓ solve for e

$$\frac{9}{4} = \frac{9}{4}e$$

$e = 1$ Value of e A1 titution

(b) Since the value of e is 1, the collision is perfectly elastic and no kinetic energy is lost. dB1

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. A plane is inclined to the horizontal at an angle α, where $\tan\alpha = \frac{3}{4}$

A particle P is held at rest at a point A on the plane.

The particle P is then projected with speed $25\,\text{m}\,\text{s}^{-1}$ from A, up a line of greatest slope of the plane.

In an initial model, the plane is modelled as being smooth and air resistance is modelled as being negligible.

Using this model and the principle of conservation of mechanical energy,

(a) find the speed of P at the instant when it has travelled a distance $\frac{25}{6}$ m up the plane from A.

(4)

In a refined model, the plane is now modelled as being rough, with the coefficient of friction between P and the plane being $\frac{3}{5}$

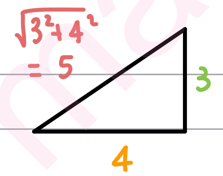
Air resistance is still modelled as being negligible.

Using this refined model and the work-energy principle,

(b) find the speed of P at the instant when it has travelled a distance $\frac{25}{6}$ m up the plane from A.

(8)

$\tan\alpha = \frac{3}{4}$



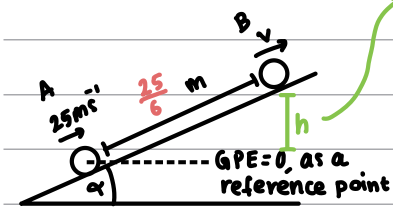
$\therefore \cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

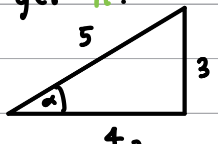
P 7 2 0 9 1 A 0 8 1 6

Question 3 continued

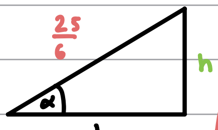
(a) Diagram



to get h:



$\tan\alpha = \frac{3}{4} \rightarrow \sin\alpha = \frac{3}{5}$



$\sin\alpha = \frac{h}{\frac{25}{6}}$

Equate:

$\frac{3}{5} = \frac{h}{\frac{25}{6}} \rightarrow h = \frac{5}{2}$ m   B1

★ conservation of mechanical energy principle: states that the total amount of mechanical energy (KE/GPE) in a closed system in the absence of dissipative forces (e.g. friction/air resistance) remains constant.

★ Remember the mechanical energy formula:

$$KE_i + GPE_i = KE_f + GPE_f$$

initial kinetic, initial grav. potential, final kinetic, final grav. potential

★ Formulae for KE and GPE:

$KE = \frac{1}{2}mv^2$ (m = mass, v = velocity)

$GPE = mgh$ (m = mass, h = change in height, $g = 9.8ms^{-2}$)

Substitute:

$\frac{1}{2}m(25)^2 + mg(0) = \frac{1}{2}mv^2 + mgh$ cancel m's   M1A1

$\frac{1}{2} \times 625 = \frac{1}{2}v^2 + g\left(\frac{5}{2}\right)$ cancel $\frac{1}{2}$

$625 - g = v^2$

$v = 24ms^{-1}$ speed   A1

Question 3 continued

(b)

★ Work-Energy Principle: an increase of KE/GPE is caused by an equal amount of positive work done on the body(e.g. engine) and a decrease of KE/GPE is caused by an equal amount of negative work done on the body(e.g. friction).

★ Remember the work-energy formulae:

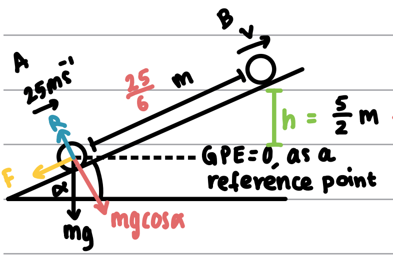
Either: WD by force + KE_i + GPE_i = KE_F + GPE_F + WD against friction

(work done, initial kinetic, initial grav. potential, final kinetic, final grav. potential, work lost to friction)

OR: WD by force + KE_i + GPE_i − WD by friction = KE_F + GPE_F

(work done, initial kinetic, initial grav. potential, we subtract this since it leaves the system as heat!, final kinetic, final grav. potential)

Diagram (include friction)



→ Since it's moving, $F_{max} = \mu R$

Using $\Sigma F_y = 0$, $R = mg\cos\alpha = \frac{4}{5}mg$ M1A1

∴ $F = \mu R$ B1

$F = \frac{3}{5} \times \frac{4}{5}mg = \frac{12}{25}mg$

→ Work done by friction is F × distance moved

∴ $WD = \frac{12}{25}mg \times \frac{25}{6} = 2mg$ J B1

Now we can substitute into WE-principle: (work done by friction) M1

$\frac{1}{2}m(25)^2 + mg(0) - 2mg = \frac{1}{2}mv^2 + mgh$ ($h = \frac{5}{2}$m from (a)) A1

(solve for v; we want v)

$\frac{625}{2}m - 2mg = \frac{1}{2}mv^2 + \frac{5}{2}mg$ cancel m's A1

$625 - 4g - 5g = v^2$ multiply both sides by 2

$625 - 9g = v^2$

$v = 23.2 ms^{-1}$ to 3sf A1

Question 3 continued

(Total for Question 3 is 12 marks)

4. A particle P of mass $2m$ kg is moving with speed $2u$ m s^{-1} on a smooth horizontal plane. Particle P collides with a particle Q of mass $3m$ kg which is at rest on the plane. The coefficient of restitution between P and Q is e. Immediately after the collision the speed of Q is v m s^{-1}

(a) Show that $v = \frac{4u(1+e)}{5}$ speed of Q after.

(6)

(b) Show that $\frac{4u}{5} \leqslant v \leqslant \frac{8u}{5}$

(2)

Given that the direction of motion of P is reversed by the collision,

(c) find, in terms of u and e, the speed of P immediately after the collision.

(2)

After the collision, Q hits a wall, that is fixed at right angles to the direction of motion of Q, and rebounds.

The coefficient of restitution between Q and the wall is $\frac{1}{6}$

Given that P and Q collide again,

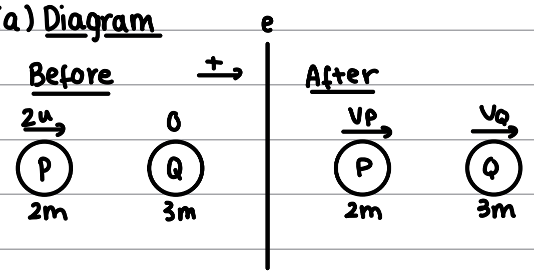
(d) find the full range of possible values of e.

(5)

P 7 2 0 9 1 A 0 1 2 1 6

Question 4 continued

(a) Diagram



We can use the conservation of linear momentum to get an equation. M1

conservation of linear momentum means: the total momentum before the collision is the same as the total momentum after.

Formula: $m_A u_A + m_B u_B = m_A v_A + m_B v_B$ (u_A: initial velocity; v_A: final velocity)

Substitute:

$$2m(2u) + 3m(0) = 2m(v_P) + 3m(v_Q) \quad \text{cancel } m\text{'s}$$

$$4u = 2v_P + 3v_Q \quad \text{Eq. 1} \quad \text{A1}$$

We can use Newton's Law of Restitution to get an equation. M1

Newton's Law of Restitution states that: when two objects collide, their speeds after the collision depend on ① speeds before the collision and ② the material from which they're made.

Formula: $e(u_A - u_B) = v_B - v_A$ (e: coefficient of restitution; u: initial speed; v: final speed)

Substitute:

$$e(2u - 0) = v_Q - v_P$$

$$2eu = v_Q - v_P \quad \text{Eq. 2} \quad \text{A1}$$

Solve simultaneously Eq1 and Eq2: use elimination method

$$4u = 2v_P + 3v_Q$$

$$2eu = v_Q - v_P \;\; (\times 2) \qquad 4eu = 2v_Q - 2v_P \quad + \quad \text{dM1}$$

$$4u + 4eu = 5v_Q$$

$$\frac{4u}{5}(1+e) = v_Q \quad \text{hence shown} \quad \text{A1}$$

(b) We know that $0 \leq e \leq 1$ ∴ M1

$$\frac{4u}{5}(1+0) \leq v \leq \frac{4u}{5}(1+1)$$

$$\therefore \frac{4u}{5} \leq v \leq \frac{8u}{5} \quad \text{hence shown} \quad \text{A1}$$

Question 4 continued

(c) Let's get V_P:

given V_Q

$$4u = 2V_P + 3\left(\frac{4}{5}u(1+e)\right)$$ → we're using the CLM equation from (a)

$$4u - \frac{12}{5}u(1+e) = 2V_P$$

$$2u - \frac{6}{5}u(1+e) = V_P$$

$$\frac{4u}{5} - \frac{6}{5}eu = V_P$$

$$\frac{2}{5}u(2-3e) = V_P$$ speed of P M1A1

(d)

As P changes direction, $V_P < 0$

$$\frac{2}{5}u(2-3e) < 0$$

$$2 - 3e < 0$$

$$\frac{2}{3} < e$$ part 1 of the inequality

To get the speed of Q after the collision with the wall, multiply by $e\left(\frac{1}{6}\right)$ and reverse the direction by multiplying by -1.

$$\therefore W_Q = -\frac{1}{6} \times V_Q$$

$$= -\frac{1}{6} \times \frac{4}{5}u(1+e)$$

$$= -\frac{2}{15}u(1+e) = W_Q$$ M1

For P and Q to collide again, W_Q must be more negative than V_P (since both are moving in the negative direction and we want the magnitude of W_Q to be larger.

$$\therefore W_Q < V_P$$ M1

$$-\frac{2}{15}u(1+e) < \frac{2}{5}u(2-3e)$$ cancel u's M1

$$-(1+e) < \frac{15}{2} \times \frac{2}{5}(2-3e)$$

$$-(1+e) < 3(2-3e)$$

$$-1 - e < 6 - 9e$$

$$8e < 7$$

A1 $$e < \frac{7}{8}$$ part 2 of the inequality

Put the 2 parts of the inequality together:

$$\frac{2}{3} < e < \frac{7}{8}$$ full range of e A1

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Question 4 continued

Question 4 continued

(Total for Question 4 is 15 marks)

TOTAL FOR FURTHER MECHANICS 1 IS 40 MARKS